

Landau-Yang Theorem and Decays of a Z' Boson into Two Z Bosons

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We study the decay of a Z' boson into two Z bosons by extending the Landau-Yang theorem to a parent particle decaying into two Z bosons. For a spin-1 parent the theorem predicts: 1) there are only two possible couplings and 2) the normalized differential cross-section depends on kinematics only through a phase shift in the azimuthal angle between the two decay planes of the Z boson. When the parent is a Z' the two possible couplings are anomaly-induced and CP-violating, respectively. At the CERN Large Hadron Collider their effects could be disentangled when both Z bosons decay leptonically.

PACS numbers: 14.70.Pw, 12.60.Cn, 11.30.Cp

Introduction – A heavy Z' boson is ubiquitous in extensions of the Standard Model (SM) [1], which frequently modify the internal and/or spacetime symmetry of the SM. The Z' could be either the gauge boson of an extra $U(1)'$ group (embedded in a larger gauge group) or the Kaluza-Klein (KK) partner of the SM Z boson propagating in extra dimensions. Its existence, if verified experimentally, has important implications beyond the mere observation of a new vector gauge boson. For example, if the Z' belongs to an extra $U(1)'$ gauge group, there must be other degree of freedom responsible for giving it a mass. Furthermore, constraints from cancellation of anomalies associated with the $U(1)'$ often require introducing new particles. On the other hand, if the Z' is the first KK mode of the SM Z boson, then additional KK modes such as the partner of other SM gauge bosons should be present as well. In other words, the discovery of a Z' boson would point us to more beyond-the-Standard-Model (BSM) particles and interactions.

Here we wish to study the coupling of a new Z' boson with two SM Z bosons. Even though the mass and couplings of a Z' could span over a wide range of parameter space, we are mainly interested in scenarios where the Z' couples to SM only through gauge boson couplings. A well-motivated example of such a possibility is in some versions of little Higgs theory with T parity [2]. To proceed, we first extend the Landau-Yang theorem [3] to decays of a parent particle into two Z bosons. The original theorem prohibits decays of a spin-1 particle into two photons. In the case of a spin-1 parent decaying into ZZ final state, simple symmetry arguments allow us to re-derive the known fact that there are only two possible couplings, which are similar to the anomalous couplings of triple neutral gauge bosons in the SM [4, 5]. In addition, we will show that the normalized differential cross section depends on kinematic variables only through a phase shift in the azimuthal angle between the two decay planes of the Z .

At the level of effective Lagrangian, the two operators contributing to the $Z'ZZ$ amplitude are both very interesting. One of them, O_A , is induced by the anomaly,

if exists, associated with the $U(1)'$. In so-called string-inspired models, O_A is present when the $U(1)'$ anomaly is cancelled either by heavy exotic fermions [6] or by an axion through the Green-Schwarz mechanism [7]. In a general non-linear sigma model (nl σ m), O_A can arise from the Wess-Zumino-Witten (WZW) term as emphasized in [8]. (One cannot resist emphasizing the obvious: the existence of WZW term depends on the UV completion of the nl σ m; see Ref. [9] for realizations with unbroken T parity.) The second operator, O_{CPV} , on the other hand is CP-violating and could arise from a *scalar* triangle loop in a two-Higgs-doublet model [10]. (O_A is P-violating, but CP-conserving.) It is possible that O_A and O_{CPV} interfere with each other, leading to potentially large CP-violating effects.

Using results from the generalized Landau-Yang theorem, it is possible to disentangle the effects of O_A and O_{CPV} in experiments by measuring the phase shift in the azimuthal angle of $Z' \rightarrow ZZ \rightarrow 4\ell$. (Previous studies focus on only one operator, but not the presence of both. For example, see Refs. [11, 12] for studies on O_A alone.) Since the method does not rely on knowing the incoming beam axis, it allows us to consider general cases when the Z' is not directly produced but is part of a long decay-chain. Experimentally, the four leptons final state is very distinct and well-studied since Higgs boson $\rightarrow ZZ \rightarrow 4\ell$ is one of the golden channels for Higgs boson discovery.

Generalized Landau-Yang Theorem – The Landau-Yang theorem uses general principle of rotation invariance and space inversion to derive certain selection rules governing decays of a parent particle into two photons. For a parent with spin less than two, the polarization state of the two photons is completely fixed by the selection rule. When it comes to ZZ final state, the only difference is the Z boson has one extra polarization state than the photon due to its massive nature. We will choose the coordinate system to be the rest frame of the parent particle in which Z_1 moves in the $+z$ direction and Z_2 in the $-z$ direction. (See Fig. 1.) Secondary decays of the Z boson are described in the rest frames of Z_1 and Z_2 , obtained by boosting along the $+z$ and $-z$ directions,

respectively. In our coordinate system, the helicity state of the two Z 's are defined as

$$\epsilon_0^{(1)} = \gamma(\beta, 0, 0, 1) = \frac{m_Y}{2m_Z}(\beta, 0, 0, 1), \quad (1)$$

$$\epsilon_0^{(2)} = \gamma(-\beta, 0, 0, 1) = \frac{m_Y}{2m_Z}(-\beta, 0, 0, 1), \quad (2)$$

$$\epsilon_{\pm}^{(1)} = (0, \mp 1, -i, 0)/\sqrt{2} = \epsilon_{\mp}^{(2)}, \quad (3)$$

where m_Y is the mass of the parent particle Y . We follow closely the notation and convention of [5] where $\epsilon_{0123} = -\epsilon^{0123} = 1$. Also notice that we have chosen both of the longitudinal polarizations to be along $+z$, even though the direction of motion is opposite. In the end there are nine possible polarization states for the ZZ , $\Psi^{\lambda_1\lambda_2}$, where $\lambda_{1,2} = +, -, 0$.

With the above definitions, it is straightforward to work out the action on $\Psi^{\lambda_1\lambda_2}$ under the following three symmetry transformations: 1) \mathcal{R}^ψ is the rotation around the z axis by an angle ψ , 2) \mathcal{R}^ξ is the rotation around the x axis by π , and 3) P is the space inversion. In effect \mathcal{R}^ψ imposes angular momentum conservation along the z axis and \mathcal{R}^ξ enforces the Bose symmetry. The selection rule for a parent with spin $J \leq 1$ and parity $P = \pm$ is summarized in Table I.

We would like to focus on a $J = 1$ parent particle and denote the helicity amplitude of $Y(\kappa) \rightarrow Z_1(\lambda_1)Z_2(\lambda_2)$ as $\mathcal{M}_{\kappa,\lambda_1\lambda_2}$, where κ is the spin projection of the parent along the $+z$ axis. (We define $\epsilon_{\pm}^{(Y)} = \epsilon_{\pm}^{(1)}$ and $\epsilon_0^{(Y)} = (0, 0, 0, 1)$.) Immediately we see that the only non-vanishing amplitudes are $\mathcal{M}_{+,+0}$, $\mathcal{M}_{+,0-}$, $\mathcal{M}_{-,-0}$, and $\mathcal{M}_{-,-0+}$. In particular, amplitudes $\mathcal{M}_{0,\pm\pm}$ and $\mathcal{M}_{0,00}$ are forbidden by the Bose symmetry, whereas all others vanish due to angular momentum conservation. It is interesting to further consider the action of the non-vanishing amplitudes under \mathcal{R}^ξ and P :

$$\mathcal{R}^\xi : \mathcal{M}_{+,+0} \leftrightarrow -\mathcal{M}_{-,-0+}, \quad \mathcal{M}_{+,0-} \leftrightarrow -\mathcal{M}_{-,-0}; \quad (4)$$

$$P : \mathcal{M}_{+,+0} \leftrightarrow -\mathcal{M}_{+,0-}, \quad \mathcal{M}_{-,-0} \leftrightarrow -\mathcal{M}_{-,-0+}. \quad (5)$$

The minus sign is due to the fact that $\epsilon_0^{(1,2)}$ are obtained from boosting $\hat{z} = (0, 0, 0, 1)$ in the direction of the respective motion of the Z . Therefore under \mathcal{R}^ξ and P , $\hat{z} \rightarrow -\hat{z}$ and $\epsilon_0^{(1)} = \gamma(\beta, 0, 0, 1) \rightarrow -\epsilon_0^{(2)}$. Moreover the spin-projection of the parent particle remains unchanged under P . One important implication of Eq. (4) is there are only two independent helicity amplitudes for any spin-1 particle decaying into two Z bosons. On the other hand, the observation that a vector boson is odd under charge conjugation (C) implies all the P -odd amplitudes should

TABLE I: Helicity states $\Psi^{\lambda_1\lambda_2}$ of the Z bosons

$P \setminus J$	0	1
+	$\Psi^{++} + \Psi^{--}, \Psi^{00}$	$\Psi^{+0} - \Psi^{0-}, \Psi^{0+} - \Psi^{0-}$
-	$\Psi^{++} - \Psi^{--}$	$\Psi^{+0} + \Psi^{0-}, \Psi^{0+} + \Psi^{0-}$

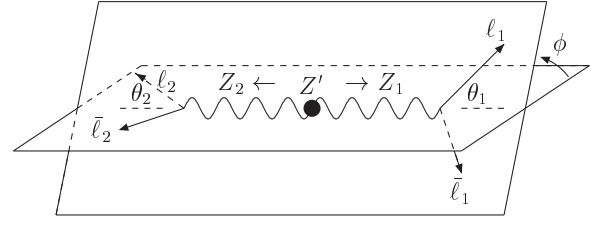


FIG. 1: Two decay planes of $Z_1 \rightarrow \ell_1 \bar{\ell}_1$ and $Z_2 \rightarrow \ell_2 \bar{\ell}_2$ define the azimuthal angle $\phi \in [0, 2\pi]$ which rotates ℓ_2 to ℓ_1 in the transverse view. The polar angles θ_1 and θ_2 shown are defined in the rest frame of Z_1 and Z_2 , respectively.

be CP -conserving and real, whereas the P -even amplitudes are CP -violating and purely imaginary. Therefore we can parametrize the four non-vanishing amplitudes as follows:

$$\begin{aligned} \mathcal{M}_{+,+0} &= A + iB = Ce^{i\delta} = -\mathcal{M}_{-,-0+}, \\ \mathcal{M}_{+,0-} &= A - iB = Ce^{-i\delta} = -\mathcal{M}_{-,-0}. \end{aligned} \quad (6)$$

The parameter C is an overall normalization and will drop out in the normalized differential cross section. The phase $\delta = \tan^{-1}(B/A)$ is 0 for $B = 0$ and $\pi/2$ for $A = 0$.

To see how δ enters into the angular distributions when $Z_1 Z_2$ further decay, recall that Z_i produces an angular dependence $\exp(im_i \phi_i)$, where $m_i = \pm, 0$ is the spin-projection and ϕ_i is the azimuthal angle in the rest frame of the Z_i . Obviously only the relative angle ϕ is physical and we can set $\phi_2 = 0$ and $\phi = \phi_1$. Then δ only enters as a phase shift in $\phi \rightarrow \phi + 2\delta$. For example, focusing only on the ϕ dependence,

$$|a_1 \mathcal{M}_{+,+0} e^{i\phi} + a_2 \mathcal{M}_{+,0-}|^2 \sim |a_1 e^{i(\phi+2\delta)} + a_2|^2, \quad (7)$$

and similarly for $\mathcal{M}_{-,\lambda_1\lambda_2}$. This argument also makes it clear that the angular distribution has the form

$$\frac{dN}{Nd\phi} \sim c_1 + c_2 \cos(\phi + 2\delta). \quad (8)$$

It is worth noting that δ is the only place where the kinematics of the system matters; all coefficients in the differential rate must then be determined by the symmetry. Furthermore, a measurement on δ could determine the relative strength between the CP -conserving and CP -violating amplitudes.

Angular Distributions – Now we turn to the specific interactions between a Z' and two Z bosons. The effective Lagrangian, when all particles are on-shell, includes only two operators at dim-4:

$$O_{CPV} = f_4 Z'_\mu (\partial_\nu Z^\mu) Z^\nu, \quad O_A = f_5 \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu (\partial_\rho Z_\sigma). \quad (9)$$

In momentum space the form factor for $Z'(q_1 + q_2, \mu) \rightarrow Z(q_1, \alpha)Z(q_2, \beta)$ can be written as

$$\Gamma_{Z' \rightarrow Z_1 Z_2}^{\mu\alpha\beta} = if_4(q_2^\alpha g^{\mu\beta} + q_1^\beta g^{\mu\alpha}) + if_5 \epsilon^{\mu\alpha\beta\rho} (q_1 - q_2)_\rho. \quad (10)$$

The non-vanishing helicity amplitudes are

$$\begin{aligned}\mathcal{M}_{+,+0} &= -\mathcal{M}_{-,0+} = R(-f_5\beta + if_4), \\ \mathcal{M}_{+,0-} &= -\mathcal{M}_{-,0-} = R(-f_5\beta - if_4),\end{aligned}\quad (11)$$

where $\beta^2 = 1 - 4m_Z^2/m_{Z'}^2$, and $R = \frac{\beta m_{Z'}}{2m_Z}$. In this case, the phase $\delta = \tan^{-1}(-f_4/f_5\beta)$.

The helicity state of the Z boson manifests itself in the angular distribution of the subsequent decay products. Particularly in the purely leptonic decays $Z_1 Z_2 \rightarrow (\ell\bar{\ell})_1 + (\ell\bar{\ell})_2$ all kinematics can be measured precisely. The polar angle θ_i is measured from the direction of motion of Z_i in the rest frame of Z' to the negatively charged lepton ℓ_i in the Z_i rest frame, whereas the azimuthal angle ϕ is measured from ℓ_2 to ℓ_1 , as illustrated in Fig. 1. In the end the differential cross section can be obtained from the expression,

$$\sum_{\kappa, h_1, h_2} \left| \sum_{\lambda_1, \lambda_2} \mathcal{M}_{\kappa, \lambda_1 \lambda_2} g_{h_1} f_{\lambda_1}^{h_1}(\theta_1, \phi) g_{h_2} f_{\lambda_2}^{h_2}(\theta_2, 0) \right|^2 \quad (12)$$

where h stands for the chirality (+, − for R, L) of the lepton with the coupling g_h to the Z . The spin-1 rotation matrix elements f_m^h are

$$\begin{aligned}2f_m^h(\bar{\theta}, \bar{\phi}) &= (1 + m h \cos \bar{\theta}) e^{im\bar{\phi}}, \\ \sqrt{2}f_0^h(\bar{\theta}, \bar{\phi}) &= h \sin \bar{\theta}.\end{aligned}\quad (13)$$

Note that m in f_m^h refers to the spin-projection $m = \pm$ and $\{\bar{\theta}, \bar{\phi}\}$ are defined with respect to the $+z$ axis in the canonical way. Since Z_2 moves in the $-z$ axis in the rest frame of the Z' , and θ_2 is defined from $-\hat{z}$ to ℓ_2 , $f_{m_2}^{h_2}$ in Eq. (12) should really be $f_{-\lambda_2}^{h_2}(\pi - \theta_2, 0)$, which equals $f_{\lambda_2}^{h_2}(\theta_2, 0)$ by Eq. (13). In the end the normalized angular distribution is

$$\begin{aligned}\frac{8\pi dN}{Nd \cos \theta_1 d \cos \theta_2 d\phi} &= \frac{9}{8} \left[1 - \cos^2 \theta_1 \cos^2 \theta_2 \right. \\ &\quad - \cos \theta_1 \cos \theta_2 \sin \theta_2 \sin \theta_1 \cos(\phi + 2\delta) \\ &\quad \left. + \frac{(g_L^2 - g_R^2)^2}{(g_L^2 + g_R^2)^2} \sin \theta_1 \sin \theta_2 \cos(\phi + 2\delta) \right].\end{aligned}\quad (14)$$

We see explicitly that all coefficients in the distribution are independent of kinematic variables such as $m_{Z'}$ and m_Z ; the only dependence on kinematics comes in through the phase shift δ . As we have seen in the previous section, this is a consequence of the Landau-Yang theorem as the symmetry of the system completely fixes the numerical coefficients. Clearly, the ϕ dependence reveals the relative magnitude of the two operators – a purely anomaly vertex O_A gives $\delta = 0$ while a purely CP-violating vertex O_{CPV} has $\delta = \pi/2$. It may seem surprising that we can observe a CP-violating operator without the presence of CP-conserving interactions, since usually one needs interference effect to observe CP violation. In our case, it is worth noting that the phase shift in the azimuthal angle ϕ is in fact an interference effect between two different helicity amplitudes, as can be seen clearly from Eq. (7).

If we integrate over both polar angles θ_1 and θ_2 , the ϕ dependence is highly suppressed by the approximate relation $g_L \approx -g_R$ for leptonic decays. The suppression can be understood from a partial \hat{C} symmetry as discussed Ref. [10]. However, if we only integrate polar angles $\cos \theta_1 \cos \theta_2 > 0$ or < 0 , then

$$\frac{2\pi dN_{\pm}}{Nd\phi} = \frac{1}{2} \left[1 + \left(\frac{9\pi^2 (g_L^2 - g_R^2)^2}{128 (g_L^2 + g_R^2)^2} \mp \frac{1}{8} \right) \cos(\phi + 2\delta) \right] \quad (15)$$

where N_{\pm} stands for $N(\cos \theta_1 \cos \theta_2 \gtrless 0)$. Since $g_L^2 \approx g_R^2$, we will ignore the $(g_L^2 - g_R^2)^2$ term from now on.

We briefly remark that if one or both of the Z 's decay hadronically, a larger sample of events could be collected. In this case a complication arises because previously we relied on the negatively charged lepton to define the polar and azimuthal angles. However we could still use the *forward jet*, defined as the jet with polar angle $\theta_i \in [0, \frac{\pi}{2}]$, in place of the negatively charged lepton and take into account the fact that different kinematic configurations will be in experimentally indistinguishable region. In the end we find there is still residual dependence on the azimuthal angle $\phi + 2\delta$. Nonetheless, we anticipate that QCD background will completely overwhelm the signal in these circumstances at the CERN LHC. However, at the linear collider it could be useful to apply the strategy to the hadronic decay of the Z .

Measurements at the LHC – Here we estimate the production rate of the two Z 's from the Z' decay that is needed to distinguish O_A and O_{CPV} , taking into account background at the LHC. Because of the QCD background, we focus on the leptonic decays $Z' \rightarrow ZZ \rightarrow 4\ell$. The main background is the SM ZZ production $q\bar{q} \rightarrow ZZ$ through a t -channel diagram which has a cross section of about 15 pb. Assuming a small decay width for the Z' , at the LHC the width measurement will be dominated by the detector energy resolution. Based on simulations using MADEVENT [13], followed by showering and hadronization in PYTHIA [14] and detector simulation in PGS4 [15], we find that the Z' width from smearing is about 12 GeV for a 240 GeV mass. Therefore we can put an invariant mass cut on the ZZ final state, $234 \text{ GeV} < m_{ZZ} < 246 \text{ GeV}$, to reduce the SM ZZ background to 79 fb.

If we see a resonance in $ZZ \rightarrow 4\ell$ above the SM background at the LHC, the first question is to determine the spin and CP property of the new particle. Such a question has been studied previously in the context of Higgs discovery. We expect that the spin of the new particle can be determined unambiguously by various proposals [16]. Previous studies, however, assumed a definite CP property of the new particle whereas presently we are mainly interested in the CP-hybrid case: the presence of both O_A and O_{CPV} . For a 5σ discovery of Z' through its decay to $ZZ \rightarrow 4\ell$, we need the ratio of the signal S to the statistical error in the background \sqrt{B} to be 5. For 100 fb^{-1} luminosity at the LHC, we find the required production for ZZ is 67 fb for a 240 GeV Z' .

The next step after the discovery is measurement of the azimuthal angular distribution and the phase shift δ . If we include the SM background and assume it has a flat distribution, the expected distribution becomes

$$n_{\pm}(\phi) \equiv \frac{dN_{\pm}}{d\phi} = \frac{N}{4\pi} \left[1 \mp \frac{1}{8} \frac{S}{S+B} \cos(\phi + 2\delta) \right]. \quad (16)$$

A Bayesian method can be used to fit the phase δ with its central value and statistical error. At this stage we will be content with a simple event counting to estimate the required production rate of Z' in order to distinguish O_A from the operator O_{CPV} . We define an “up-down asymmetry” \mathcal{A}_{ud} in the absence of background as

$$\left(\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2} \right) \frac{n_+(\phi) - n_-(\phi)}{N} d\phi = -\frac{\cos(2\delta)}{4\pi}. \quad (17)$$

For operator O_A only ($\delta = 0$), $\mathcal{A}_{ud} = -\frac{1}{4\pi}$, whereas $\mathcal{A}_{ud} = \frac{1}{4\pi}$ for O_{CPV} only ($\delta = \pi/2$). If we would like to discriminate O_A from O_{CPV} at 99.7% confidence level (3σ), we would require the difference in the number of the “up-down” asymmetrical events $S_A = \mathcal{A}_{ud} \times S$ over the statistical errors of the total number of events $\sqrt{S+B}$ to be 3:

$$\frac{|S_A(\delta = 0) - S_A(\delta = \pi/2)|}{\sqrt{S+B}} = \frac{S}{2\pi\sqrt{S+B}} = 3. \quad (18)$$

Then the required production rate of the Z boson from Z' decay is 0.9 pb for a 240 GeV Z' .

Such a production rate could in fact be easily fulfilled in models where the branching ratio of $Z' \rightarrow ZZ$ is not very small. For instance, in the lightest Higgs model with anomalous T parity, the lightest T-odd particle B_H (the Z') will only decay into W^+W^- and ZZ through the

WWZ term, which gives rise to a non-zero O_A , and the branching ratio of $Z' \rightarrow ZZ$ turns out to be roughly 1/3 [11]. Since the T-odd particle is always pair-produced and eventually decays into B_H , the required B_H production rate, which is also the total cross section for T-odd particles, will be 1.3 pb. If we choose a typical parameter $f = 1.5$ TeV, $m_{B_H} = 240$ GeV, we find that even the T-quark production channel alone with a T-quark mass 750 GeV will give us the necessary production rate for discrimination [17].

Conclusion – In this work we study decays of a new Z' gauge boson into two SM Z bosons by extending the Landau-Yang theorem to a parent particle decaying into two Z bosons. The original theorem forbids decays of a spin-1 particle into two photons based on simple symmetry arguments. We show the generalized Landau-Yang theorem makes strong predictions on the polarization states of the two Z 's and the differential distribution of its subsequent decays. In particular, the normalized differential cross section depends on kinematics only through a phase shift in the azimuthal angle between the two decay planes of the Z . For a Z' boson, the two couplings are anomaly-induced and CP-violating, whose effects could be disentangled by measuring the phase shift at the CERN LHC. The method could potentially be applied to the case of a scalar decaying into ZZ final state which does not have a definite CP property.

Acknowledgements – This work was supported in part by the U.S. Department of Energy under DE-FG02-84ER40173 (W.-Y. K.) and DE-AC02-06CH11357 (I. L. and J. S.). J. S. was also supported by the University of Chicago under section H.28 of its contract W-31-109-ENG-38 to manage Argonne National Laboratory. W.-Y. K. also thanks NCTS, Taiwan for hospitality.

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